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Tunneling Effect of Two Horizons from a Reissner-Nordstrom Black Hole

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Abstract The understanding of possible role played by the inner horizon of black holes in black hole thermodynamics is still somewhat incomplete. By adopting Damour-Ruffini method and the thin film model which is developed on the base of brick wall model suggested by 't Hooft, we calculate the temperature and the entropy of the inner horizon of a R-N black hole. We conclude that the temperature of inner horizon is positive and the entropy of the inner horizon is proportional to the area of the inner horizon. In addition, the cut-off factor is 90β, which is same in calculation of the entropy of the outer horizon. So, we prove the existence of thermal characters of the inner horizon. Then, we discuss that if the contribution of the inner horizon is taken into account to the total entropy of the black hole, the Nernst theorem can be satisfied. At last, we study the tunneling effect including the inner horizon of the Reissner-Nordstrom black hole. We calculate the tunneling rate of the outer horizon Γ_+ and the inner horizon Γ_-. The total tunneling rate Γ should be the product of the rates of the outer and inner horizon, $\Gamma = \Gamma_+ \cdot \Gamma_-$. We find that the total tunneling rate is in agreement with the Parikh's standard result, $\Gamma \to \exp(\Delta S_{BH})$, and there is no information loss.

Keywords Entropy · Tunneling effect · Inner horizon

1 Introduction

Since Bekenstein suggested that the entropy of a black hole is proportional to its surface area, the concerned research work has gotten much progress [1, 4]. However, our understanding of the inner horizon is still somewhat incomplete.

In this paper, by adopting Damour-Ruffini method [2], we calculate the temperature of the inner horizon of the R-N black hole and prove that the inner horizon exists thermal effect. The temperature is positive and is a constant all over the inner horizon. To the R-N space time, the Killing vector field $(\frac{\partial}{\partial t})^a$ both outside the outer horizon and inside the inner

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horizon is time-like, where the space-time is stationary. So, the surface gravity on the inner horizon can be well defined. Using the formula in the Ref. [14], we gain the result that κ_- is negative, unlike the surface gravity $\kappa_+ > 0$ on the outer horizon. This can be understood easily. The outer horizon is a future horizon to an observer outside it. The matter can only fall into the black hole and can not escape from it in a classical situation. However, the inner horizon is a past horizon to a observer inside it. The observer inside the inner horizon regards it as a horizon of a white hole. The white hole can emit matter classically, not absorb matter. So, the surface gravity of outer horizon, κ_+, is directed to the horizon and is positive; κ_-, is directed to the singularity, not to the inner horizon. Hence, it is reasonable that the surface gravity of the inner horizon is negative. The effect of the inner horizon absorbing black hole body radiation can be regarded as Hawking absorbtion [27]. We can explain the Hawking radiation of the R-N black holes as follows. A flow of positive energy particles produced near the singularity propagates in time and reach the inner horizon. Then, these particles are scattered by the inner horizon and travel in the reversed time toward the outer horizon where they are scattered again. At last, they travel forward in time to infinity.

We calculate the entropy of the inner horizon by using the thin film brick wall model [6] which is on the base of brick wall model proposed by t' Hooft [11]. Because the entropy is associated with the field in the small region where exists the local thermal equilibrium and the statistical laws are valid [12]. So, the field outside the outer horizon can be supposed to non-zero only in the thin film $(r_+ + \varepsilon) \to (r_+ + \varepsilon + \delta)$, where r_+ is the radius of the outer horizon, ε is the ultraviolet cut-off and δ is the thickness of the thin film. Using this model we can work out the entropy of the outer horizon. There exists a time-like Killing vector field in the region $r < r_-$. The field in the thin film $(r_- - \varepsilon) \to (r_- - \varepsilon - \delta)$ can be regarded as non-zero when we calculate the entropy of the inner horizon. We obtain that the entropy of the inner horizon is also proportional to its area and the cut-off is 90β.

There is still an open problem of the entropy of the black hole [3, 13, 15]. According to Nernst theory of the third law of ordinary thermodynamics, the entropy of a system must go to zero as its temperature reaches zero. If this assertion is used to black holes, we find that the entropy of the black hole with two horizons, such as Kerr black hole, does not go to zero as its temperature approaches absolute zero [5, 16]. If the black hole with two horizons is regarded as a thermodynamics system composed of two subsystem, the outer horizon and inner horizon, the entropy of the black hole should include the contribution both of the outer and inner horizon [25]. In this paper, As an example of the black holes with two horizon, we propose that the entropy of the R-N black hole can be written as:

$$S_{BH} = S_+ + S_-, \tag{1}$$

where, S_+ and S_- are the entropy contributed by the outer and inner horizon respectively. Then, when the temperature of the R-N black hole approaches zero, the total redefined entropy, $S_{BH} = S_+ + S_-$, vanishes. The Nernst theorem is satisfied.

Hawking's discovery that the black hole can emit thermal radiation gives rise to a famous paradox—the information loss paradox of black hole physics. When Hawking first proved the existence of black hole radiation, he described it as tunneling triggered by vacuum fluctuations near the horizon. But, actual derivation of Hawking radiation did not proceed in this way at all and there did not seem to be any barrier.

Recently, Parikh and Wilczek gave an enlightening suggestion that Hawking radiation could be treated as a tunneling precess [7–9]. The self-interaction effect is taken into account in their method. They obtained a leading correction to the emission rate arising from loss of mass of the black hole and concluded that the information was conserved. Following this method, Zhang and Zhao have extended Parikh's method from static black holes to the non-spherical symmetric stationary black holes and radiation of charged particle and massive particle and made much progress [18, 21–23]. However, the information is not conserved if we only consider the tunneling process of the outer horizon after redefining the entropy of the R-N black hole. In this paper, we have a new idea that the tunneling effect of the inner horizon must be taken into account because there do exist thermal characters of the inner horizon. A positive energy particle created near the singularity travels forward in time and arrives at the inner horizon where the tunneling rate is Γ_-. Then, it goes in the reversed time toward the outer horizon. The tunneling rate is Γ_+ at the outer horizon. The total tunneling rate Γ should be the product of the rates of the outer and inner horizon, $\Gamma = \Gamma_+\Gamma_-$. Our result is in agreement with the Parikh's standard result, $\Gamma \to \exp(\Delta S_{BH})$, and the information is conserved, where S_{BH} is the sum of the contribution of the outer and inner horizon.

Throughout the paper, the units $G = c = \hbar = k_B = 1$ are used.

2 Temperature of the Inner Horizon

The line element of R-N black hole is described by

$$ds^2 = -\frac{(r-r_+)(r-r_-)}{r^2}dt^2 + \frac{r^2}{(r-r_+)(r-r_-)}dr^2 + r^2 d\Omega^2, \tag{2}$$

where r_+ and r_- are the radiuses of the event horizon and the inner horizon, respectively. r_+ and r_- satisfy

$$r_\pm = m \pm \sqrt{m^2 - Q^2}. \tag{3}$$

For the line-element (2), there exists a time-like Killing vector field ξ^μ. Substituting $I^\mu = \xi^\mu$ into the definition of the surface gravity [14]

$$\kappa^2 = -\frac{1}{2}(\nabla^a I^b)(\nabla_a I_b), \tag{4}$$

we obtain [14, 24]

$$\kappa = -\frac{1}{2}\left(\sqrt{\frac{g^{11}}{-g_{00}}}\frac{dg_{00}}{dr}\right)_{r=r_H}. \tag{5}$$

The above equation can be used to calculate the surface gravity of the outer horizon, κ_+. Because $(\frac{\partial}{\partial t})^a$ is a time-like Killing vector field in the region, $r < r_-$, (5) is suitable to the surface gravity of the inner horizon, κ_-. So, we obtain

$$\kappa_+ = \frac{r_+ - r_-}{2r_+^2}, \tag{6}$$

$$\kappa_- = -\frac{r_+ - r_-}{2r_-^2}. \tag{7}$$

Note κ_- is negative. It is because that the surface gravity of the inner horizon is directed to the singularity, not to the horizon, unlike κ_+ which is directed to the outer horizon. We can rewrite the (2) as

$$ds^2 = \frac{(r-r_+)(r-r_-)}{r^2}\left[-dt^2 + \frac{r^4}{(r-r_+)^2(r-r_-)^2}dr^2\right] + r^2 d\Omega^2. \tag{8}$$

Let

$$dr_* = \frac{r^2}{(r-r_+)(r-r_-)}dr, \tag{9}$$

we can choose r_* as

$$r_* = r + \frac{1}{2\kappa_+}\ln\left(\frac{|r-r_+|}{r_+}\right) - \frac{1}{2|\kappa_-|}\ln\left(\frac{|r-r_-|}{r_-}\right). \tag{10}$$

If we do the differential of the above equation, we can obtain (9). We will calculate the temperature of the inner horizon as below adopting Damour-Ruffini method and illustrate that it is positive.

In 1976, Damour and Ruffini suggested a method which can demonstrate the Hawking radiation [2]. The Klein-Gordon equation is

$$(\Box - \mu^2)\Phi = 0. \tag{11}$$

Making the separation of variables as, $\Phi = Y(\theta,\varphi)\psi(t,r)$, the radical equation can be written as

$$g^{00}\frac{\partial^2\psi}{\partial t^2} + g^{11}\frac{\partial^2\psi}{\partial r^2} + \left[\aleph(r) + \frac{2r}{r^2}g^{11}\right]\frac{\partial\psi}{\partial r} + \frac{1}{r^2}[l(l+1)\psi] - \mu^2\psi = 0, \tag{12}$$

where $\aleph = \frac{r-r_+}{r^2} + \frac{r-r_-}{r^2} - \frac{2r(r-r_+)(r-r_-)}{r^4}$.

Introducing the tortoise coordinate transformation (9), the radical equation becomes

$$\frac{\partial^2\psi}{\partial t^2} + \frac{g^{11}}{g^{00}}\left\{\frac{r^4}{(r-r_+)^2(r-r_-)^2}\frac{\partial^2\psi}{\partial r_*^2} + \left[\frac{2r}{(r-r_+)(r-r_-)} - \frac{r^2(2r-r_+-r_-)}{(r-r_+)^2(r-r_-)^2}\right]\frac{\partial\psi}{\partial r_*}\right\}$$
$$+ \frac{1}{g^{00}}\left[\aleph(r) + \frac{2r}{r^2}g^{11}\right]\frac{r^2}{(r-r_+)(r-r_-)}\frac{\partial\psi}{\partial r_*} + \frac{1}{g^{00}}\left[\frac{l(l+1)}{r^2} - \mu^2\right]\psi = 0. \tag{13}$$

When $r \to r_\pm$, $(g^{00})^{-1} = g_{00} = -g^{11} \to 0$, and

$$\aleph \to \frac{r_+ - r_-}{r^2}, \quad r \to r_+, \tag{14}$$

$$\aleph \to -\frac{r_+ - r_-}{r^2}, \quad r \to r_-, \tag{15}$$

so, when $r \to r_\pm$, the radical equation (13) can be reduced to the standard form of the wave equation,

$$\frac{\partial^2\psi}{\partial t^2} - \frac{\partial^2\psi}{\partial r_*^2} = 0, \tag{16}$$

which shows that there are waves which propagate radically near the outer and inner horizon. It is well known that Hawking radiation exists near the outer horizon. In this paper, we are only interested in the case near the inner horizon ($r < r_-$). Introducing the retarded Eddington-Finkelstein coordinate $u = t - r_*$ [25], the solutions near the inner horizon are

$$\psi_{out} = \exp(-i\omega t + i\omega r_*) = \exp(-i\omega u), \tag{17}$$

$$\psi_{in} = \exp(-i\omega t - i\omega r_*) = \exp(-i\omega u - 2i\omega r_*). \tag{18}$$

From the above equations, it is obvious that $r_* \to -\infty$ as $r \to r_-$; $r_* \to 0$ as $r \to 0$. So, (17) and (18) represent the outgoing and ingoing waves respectively. This is unlike the case that the advanced Eddington-Finkelstein coordinate is used when we calculate the temperature of the outer horizon. The reason is that an observer who is located inside the inner horizon regards the inner horizon as a past horizon, not as a future horizon. Note that the ingoing wave propagates towards the inner horizon. As $r \to r_-$,

$$r_* \to \frac{1}{2\kappa_-} \ln(r_- - r). \tag{19}$$

Thus, the ingoing wave can be written as

$$\psi_{in} = e^{-i\omega u}(r_- - r)^{-\frac{i\omega}{\kappa_-}}. \tag{20}$$

We can find that ψ_{in} is not analytic at the inner horizon. Hence, ψ_{in} should be extended analytically around the inner horizon r_- along the upper semicircle of radius $|r - r_-|$ in the complex r plane [25], into the one-way membrane region between the inner and outer horizon, as $|r_- - r|e^{-i\pi} = (r - r_-)e^{-i\pi}$. So, ψ_{in} in the region $r_- < r < r_+$ can be written as

$$\begin{aligned}\psi_{in} &= e^{-i\omega u}[(r - r_-)e^{-i\pi}]^{-\frac{i\omega}{\kappa_-}} = [e^{-i\omega u}(r - r_-)^{-i\omega/\kappa_-}][e^{-\pi\omega/\kappa_-}] \\ &= \psi'_{in}(r - r_-)e^{-\pi\omega/\kappa_-},\end{aligned} \tag{21}$$

where

$$\psi'_{in} = e^{-i\omega u}(r - r_-)^{-i\omega/\kappa_-} = e^{-i\omega u}e^{-2i\omega r_*}. \tag{22}$$

The total ingoing wave function can be written in a uniform form

$$\psi = N_\omega[Y(r_- - r)\psi_{in}(r_- - r) + e^{-\pi\omega/\kappa_-}Y(r - r_-)\psi'_{in}(r - r_-)], \tag{23}$$

where

$$Y(r) = \begin{cases} 1, & r \geq 0 \\ 0, & r < 0. \end{cases} \tag{24}$$

According to the normalization condition

$$(\psi, \psi) = \pm 1. \tag{25}$$

So, the thermal spectrum and temperature of the inner horizon are, respectively,

$$N_\omega^2 = \frac{1}{e^{\frac{\omega}{T_-}} \pm 1}, \tag{26}$$

$$T_- = \frac{-\kappa_-}{2\pi}. \tag{27}$$

It is shown that the temperature of the inner horizon is positive too.

Therefore we have proved that there does exist radiation from the region $r < r_-$ to the inner horizon. The effect can be regarded as "Hawking absorbtion" [26, 27]. The outer horizon is in thermal equilibrium with the thermal radiation outside the black hole. Similarly, the inner horizon is in thermal equilibrium with the thermal radiation inside the inner horizon. The inner horizon absorbs thermal radiation at temperature T_-, and at the same time it emits thermal radiation at temperature T_-. So, the inner horizon is a thermal system with temperature T_-. We can explain Hawking radiation as follows. The positive energy particles created near the singularity are absorbed by the inner horizon. Then, they travel in the reverse time direction, transiting the "one-way membrane" region and arrive at the outer horizon. After scattered by the out horizon, they travel to infinity as Hawking radiation.

3 Entropy of the Inner Horizon

We use the thin film brick model [6, 12] to calculate the entropy of the inner horizon. Let us substitute the metric into Klein-Gordon equation, which describes the scalar field with mass μ:

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}g^{\mu\nu}\frac{\partial\Phi}{\partial x^\mu}\right)-\mu^2\Phi=0. \tag{28}$$

The solution of the equation can be written as

$$\Phi=e^{-i\omega t}\psi(r)Y_{l,m}(\theta,\varphi). \tag{29}$$

With the WKB approximation, we get the wave vector:

$$k^2=\frac{1}{(r-r_+)(r-r_-)}\left[\frac{r^4\omega^2}{(r-r_+)(r-r_-)}-\mu^2r^2-l(l+1)\right]. \tag{30}$$

According to quantum statistical mechanics, the free energy is given by

$$F=-\frac{1}{\pi}\int_0^\infty d\omega\int_{r_--\varepsilon}^{r_--\varepsilon-\delta}dr\int_l(2l+1)\frac{k}{e^{\beta\omega}-1}dl. \tag{31}$$

Studying the integration on l,ω, we get

$$F=-\frac{2\pi^3}{45\beta^4}\int_{r_--\varepsilon}^{r_--\varepsilon-\delta}\frac{r^6}{(r-r_+)^2(r-r_-)^2}dr \tag{32}$$

$$=-\frac{2\pi^3}{45\beta^4}\frac{r_-^6}{(r_+-r_-)^2}\int_{r_--\varepsilon}^{r_--\varepsilon-\delta}\frac{dr}{(r-r_-)^2} \tag{33}$$

$$=\frac{2\pi^3}{45\beta^4}\frac{r_-^6}{(r_+-r_-)^2}\frac{\delta}{\varepsilon(\varepsilon+\delta)}. \tag{34}$$

Considering the temperature of the inner horizon $\frac{1}{\beta}=T_-=\frac{r_+-r_-}{4\pi r_-^2}$, the entropy is

$$S_-=\beta^2\frac{\partial F}{\partial\beta}=-\frac{\pi r_-^2}{90\beta}\frac{\delta}{\varepsilon(\varepsilon+\delta)}. \tag{35}$$

Selecting appropriate cut-off ε and δ as $\frac{\delta}{\varepsilon(\varepsilon+\delta)} = 90\beta$, the entropy of the inner horizon is

$$S_- = -\frac{1}{4}A_-, \tag{36}$$

where A_- is the area of the inner horizon. The entropy is also proportional to the area of the inner horizon and cut off is 90β which is the same as the cut off in the calculation of the entropy of the outer horizon. The above result and the temperature of the inner horizon satisfy the familiar formula $\frac{1}{T} = \frac{dS_{BH}}{dm}$:

$$T_- = \left(\frac{dS_-}{dm}\right)^{-1} = -\frac{\kappa_-}{2\pi} = \frac{r_+ - r_-}{4\pi r_-^2}. \tag{37}$$

The reason why the entropy of the inner horizon is negative is not clear. It is an open question.

The Nernst theorem claims that the entropy of a system must go to zero as its temperature reaches zero. Previously, the entropy and the temperature of the R-N black hole are, $S_{BH} = \pi r_+^2$ and $T = \frac{\kappa_+}{2\pi}$, respectively. It is obvious that the entropy does not vanish when the temperature, $T \to 0$. If the total entropy of the black hole can be regarded as the sum of the contribution of the outer and inner horizon [25]:

$$S_{BH} = S_+ + S_- = \pi r_+^2 - \pi r_-^2 = \pi(r_+^2 - r_-^2), \tag{38}$$

it is manifest that when the temperature, $T = \frac{r_+ - r_-}{4\pi r_+^2}$, goes to absolute zero, $r_+ = r_-$, the entropy of the black hole (38) vanishes. Consequently, the Nernst theorem is satisfied.

4 Tunneling Effect

Because the entropy of the black hole is contributed not only by the outer horizon but also by the inner horizon, the tunneling effect of the inner horizon must be considered. Otherwise, the information will not be conserved. The total tunneling rate should be the product of the tunneling rates of the outer horizon and the inner horizon. In order to simplify the calculation, we adopt the Eddington coordinate to study the tunneling effect [10]. Using Eddington coordinate, the line element (2) can be rewritten as

$$ds^2 = -\frac{(r - r_+)(r - r_-)}{r^2} dv^2 + 2dvdr + r^2 d\Omega^2. \tag{39}$$

It is obvious that the components are not diverge at the outer and the inner horizon. The radial null geodesics in Eddington-Finkelstein coordinates obey

$$\dot{r} \equiv \frac{dr}{dv} = \frac{1}{2}\frac{(r - r_+)(r - r_-)}{r^2}. \tag{40}$$

Equations (39) and (40) are modified when the self-gravitation of the particle is considered [8]. We can consider the particle with energy ω as a shell of energy. We fix the total mass (ADM mass) and allow the hole mass to fluctuate. When the shell of energy ω travels on the geodesics, we should replace m with $m - \omega$ in the geodesic (40) and in the line elements (39) to describe the moving of the shell [9, 17, 18].

In the semiclassical limit, we could apply the WKB formula. This relates the tunneling amplitude to the imaginary part of the particle action at stationary phase. The emission rate, Γ, is the square of the tunneling amplitude [7, 19, 20]:

$$\Gamma \sim \exp\left(-2\,\mathrm{Im}\, I\right), \tag{41}$$

where I is the action. A positive energy particle created near the singularity travels forward in time. When it arrives at the inner horizon, the tunneling rate of the inner horizon is

$$\Gamma_- \sim \exp\left(-2\,\mathrm{Im}\, I_-\right). \tag{42}$$

The radius of the inner horizon increases when the mass of the black hole decreases. It is this expansion that sets the barrier. The imaginary part of the action for an outgoing positive energy particle which crosses the inner horizon outwards from r_{in} to r_{out} could be expressed as

$$\mathrm{Im}\, I_- = \mathrm{Im}\int_{r_{in}}^{r_{out}} p_r dr = \mathrm{Im}\int_{r_{in}}^{r_{out}}\int_0^{p_r} dp_r' dr, \tag{43}$$

where p_r is canonical momentum conjugate to r, $r_{in} = r_-$ is the initial radius of the inner horizon, and $r_{out} = r_-'$ is the final radius of the inner horizon, where $r_-' = r_-(m-\omega)$. Here it is noted that $r_-' > r_-$. We substitute Hamilton' equation $\dot{r} = \frac{dH}{dp_r}\mid_r$ into (43), change variable from momentum to energy, and switch the order of integration to obtain

$$\mathrm{Im}\, I_- = \mathrm{Im}\int_m^{m-\omega}\int_{r_{in}}^{r_{out}} \frac{dr}{\dot{r}} dH = \mathrm{Im}\int_0^{\omega}\int_{r_{in}}^{r_{out}} \frac{2r^2 dr}{(r-r_+')(r-r_-')}\left(-d\omega'\right). \tag{44}$$

We have used the modified (40) and H is the ADM energy of the space-time [7]. In (44), $r = r_-'$ is the first order pole. Do the integral r first, we obtain

$$\mathrm{Im}\, I_- = 2\pi\int_0^{\omega} \frac{r_-'^2}{r_+' - r_-'} d\omega'. \tag{45}$$

So, the tunneling rate is

$$\Gamma_- = \exp[-2\,\mathrm{Im}\, I_-] = \exp\left[-\int_0^{\omega} 4\pi\left(\frac{r_-'^2}{r_+' - r_-'}\right) d\omega'\right]. \tag{46}$$

Using the same method, the tunneling rate of the outer horizon is

$$\Gamma_+ = \exp\left[-\int_0^{\omega} 4\pi\left(\frac{r_+'^2}{r_+' - r_-'}\right) d\omega'\right]. \tag{47}$$

Thus the total tunneling rate is

$$\Gamma = \Gamma_+ \cdot \Gamma_- = \exp\left[-\int_0^{\omega} 4\pi\left(\frac{r_+'^2 + r_-'^2}{r_+' - r_-'}\right) d\omega'\right]. \tag{48}$$

Though it is rather difficult to work out the integral with respect to ω' directly, we can make the physical meaning clear as follows. The entropy (38) derivative of m is

$$\frac{\partial S_{BH}}{\partial m} = \pi\left(2r_+\frac{\partial r_+}{\partial m} - 2r_-\frac{\partial r_-}{\partial m}\right)$$

$$= 4\pi\left(\frac{r_+^2 + r_-^2}{r_+ - r_-}\right). \tag{49}$$

Doing the integral of m, (49) becomes to

$$\Delta S_{BH} = \int_m^{m-\omega} \frac{\partial S_{BH}}{\partial m'} dm' = \int_m^{m-\omega} 4\pi\left(\frac{r_+'^2 + r_-'^2}{r_+' - r_-'}\right) dm'. \tag{50}$$

Substituting $m' = m - \omega$ into above equation, we have

$$\Delta S_{BH} = -\int_0^{\omega} \frac{\partial S_{BH}}{\partial m'} d\omega' = -\int_0^{\omega} 4\pi\left(\frac{r_+'^2 + r_-'^2}{r_+' - r_-'}\right) d\omega'. \tag{51}$$

Comparing (48) with (51), we find

$$\Gamma \to \exp(-2\,\mathrm{Im}\, S) = \exp(\Delta S_{BH}). \tag{52}$$

This result is in agreement with Parikh' work.

5 Discussion and Conclusion

If the contribution of the inner horizon is taken into account to the entropy of the black hole, the Nernst theorem can be satisfied. Hence, we suggest that the black hole with two horizons is regarded as a thermodynamics system composed of two subsystem, the outer horizon and inner horizon. The entropy of the black hole should include the contribution both of the outer and inner horizon. Consequently, in the method proposed by Parikh and Wilczek, the tunneling effect of the inner horizon must be considered. We calculate the total tunneling rate including the outer and the inner horizon and find that our result is in agreement with Parikh's work. It is shown that on the base of considering the new formula of the entropy and the tunneling effect of the inner horizon, the result is still correct and there is no information loss. We believe that our method of the tunneling effect of two horizons can be used to any black hole which has two horizons. Our work of the radiation effect of the inner horizon has much importance because it supports the idea that all horizons of space-time emit radiation.

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References

1. Bekenstein, J.D.: Phys. Rev. D **7**, 2333 (1973)
2. Damour, T., Ruffini, R.: Phys. Rev. D **14**, 332 (1976)
3. Frolov, V.P., Page, D.N.: Phys. Rev. Lett. **71**, 3902 (1993)
4. Hawking, S.W.: Commun. Math. Phys. **43**, 199 (1975)
5. Lee, H., Kim, S.W., Kim, W.T.: Phys. Rev. D **54**, 6559 (1996)
6. Li, X., Zhao, Z.: Phys. Rev. D **62**, 104001 (2000)
7. Parikh, M.K.: Energy conservation and Hawking radiation. hep-th/0402166 (2004)
8. Parikh, M.K.: A secret tunnel through the horizon. hep-th/0405160 (2004)
9. Parikh, M.K., Wilczek, F.: Phys. Rev. Lett. **85**, 5042 (2000)
10. Ren, J.: Int. J. Theor. Phys. **46**(12), 3109 (2007)
11. 't Hooft, G.: Nucl. Phys. B **256**, 727 (1985)
12. Tian, G.H., Zhao, Z.: Nucl. Phys. B **636**, 418 (2002)

13. Unruh, W.G., Wald, R.W.: Phys. Rev. D **25**, 942 (1982)
14. Wald, R.W.: In: General Relativity, pp. 330–334. University of Chicago Press, Chicago and London (1984)
15. Wald, R.W.: Quantum Field Theory in Curved. Space-Time and Black Hole Thermodynamics. University of Press Chicago, Chicago (1994)
16. Wald, R.W.: The Nernst theorem and black hole thermodynamics. gr-qc/9704008 (1997)
17. Zhang, J.Y.: Modern Phys. Lett. A **22**, 1821–1828 (2007)
18. Zhang, J.Y., Fan, J.H.: Phys. Lett. B **648**, 133–138 (2007)
19. Zhang, J.Y., Fan, J.H.: Physica A **379**, 90–100 (2007)
20. Zhang, J.Y., Fan, J.H.: Chin. Phys. **16**, 3879–3884 (2007)
21. Zhang, J.Y., Zhao, Z.: Nucl. Phys. B **725**, 173–180 (2005)
22. Zhang, J.Y., Zhao, Z.: J. High Energy Phys. (10), Art. No. 055 (2005)
23. Zhang, J.Y., Zhao, Z.: Phys. Lett. B **638**, 110–113 (2006)
24. Zhao, Z.: Acta Phys. Sin. **30**, 1508 (1981)
25. Zhao, Z.: Int. J. Theor. Phys. **38**(5), 1539 (1999)
26. Zhao, Z., Gui, Y.X., Liu, L.: J. Beijing Norm. Univ. (Nat. Sie. Ed.) **19**, 73 (1983) (in Chinese)
27. Zhao, Z., Zhu, J.Y., Liu, W.B.: Chin. Phys. Lett. **16**, 698 (1999)